

## KD2 Pro Compliance to ASTM and IEEE Standards

The KD2 Pro complies fully with ASTM D5334-14. A certificate of compliance can be downloaded at [thermalresistivity.com](http://thermalresistivity.com) or can be requested directly from Decagon.

### **ASTM Standard Revised in 2014**

ASTM D5334-14 is a significantly updated version of the Standard Test Method for Determination of Thermal Conductivity of Soils and Rock by Thermal Needle Probe Procedure. It represents the best practices in accordance with current research in heat and mass transfer. For accurate measurements, it is important to specify and use the most current version of this standard.

### **Elements of Compliance to ASTM D5334-14**

Both KD2 Pro single thermal needles (the TR-1 and the KS-1) have sufficient length to diameter ratio to simulate conditions for an infinitely long, infinitely thin heating source.

The KD2 Pro includes a linear heat source and a temperature measuring element. Temperatures are measured with a resolution of 0.001 C.

The KD2 Pro produces constant current; reads voltage and current to better than the nearest 0.01 V and 0.01 ampere; measures time to better than the nearest 0.1 second.

Temperature decay with time is included in analysis to minimize effects of temperature drift. Microprocessor-based analytical methods comply with all specifications of ASTM D5334 14. We calibrate the KD2 Pro to ensure accurate measurements. Accuracy verification standard material is included.

Accessories included are capable of drilling a pilot hole with a diameter and depth equal to the dimensions of the needle.

### **IEEE 442-03**

The IEEE is considering updates to IEEE 442-03 (which was last subject to thorough consideration and revision in 1981). The KD2 complies with all theoretical assumptions upon which IEEE 442-03 is based (see Appendix A, Theory and Analysis), but makes full use of technologically superior sensors and microprocessor based analysis rather than the homemade probes and pencil-and-paper analysis methods which were in common use when IEEE was first drafted.

The IEEE states in the introduction to this standard, "Every IEEE Standard is subjected to review at least once every five years for revision or reaffirmation. When a document is more than five years old, and has not been reaffirmed, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art." IEEE 442-03 was last reaffirmed in 2003.

As the IEEE works to update this standard, it may be advisable to specify and follow ASTM 5334-14 which, due to its significant recent revision, better represents current state of the art in heat and mass transfer. Inaccuracies that may occur when explicitly following the field probe dimensions and probe heating times outlined in IEEE 442-03 are shown in Appendix A.

### **Soil Science Society of America (SSSA), Methods of Analysis Part 4 Physical Methods 5.3 (Thermal Conductivity pp 1209-1226)**

The KD2 Pro probe needle sizes, heating times, accuracy specifications, and internal data analysis follow recommendations outlined in the SSSA methods. A Certificate of Compliance is available from Decagon on request.

## Appendix A: Theory and Analysis

### Transient Line Heat Source Theory

The method used in the ASTM and IEEE thermal conductivity/resistivity measurement standards (IEEE 442 and ASTM 5334) is generally called the transient line heat source or transient heated needle method. If heat at a constant rate,  $q$  is applied to an infinitely long and infinitely small “line” source, the temperature response of the source over time can be described by the equation:

$$\Delta T = -\frac{q}{4\pi k} Ei\left(\frac{-r^2}{4Dt}\right) \quad (1)$$

where  $k$  is the thermal conductivity of the medium in which the line is buried,  $D$  is the thermal diffusivity of the medium,  $r$  is the distance from the line at which temperature is measured, and  $Ei$  is the exponential integral.  $Ei$  is defined in the following equation, and can be approximated by the series shown:

$$\begin{aligned} -Ei(-\alpha) &= \int_{\alpha}^{\infty} \left(\frac{1}{u}\right) \exp(-u) du \\ &= \frac{-\gamma - \ln \alpha + \alpha - \alpha^2}{4 + \dots} \end{aligned} \quad (2)$$

in which  $\gamma = 0.5772\dots$  is Euler’s constant and  $\alpha = r^2/4Dt$ .

The terms beyond  $\ln \alpha$  in the series expansion of  $Ei$  become negligibly small for long times when  $r$  is small and  $D$  is large, so eq. 2 can be approximated as

$$\Delta T \approx \frac{q}{4\pi k} \ln t + C \quad (3)$$

where  $C$  is a constant. Thus, if early time data are ignored, a graph of  $\Delta T$  vs.  $\ln t$  becomes a straight line with slope equal to  $q/4\pi k$ . Since two points define a straight line,  $k$  can be computed from Equation 4.

$$k = \frac{q(\ln t_2 - \ln t_1)}{4\pi(\Delta T_2 - \Delta T_1)} \quad (4)$$

The resistivity,  $\rho$ , is the reciprocal of  $k$ . IEEE 442-03 uses this simplified method to obtain  $k$ .

### Assumptions of Equation (4)

This simplified equation rests on three assumptions:

1. That the exponential integral can be approximated by the logarithm.
2. That the probe is infinitely long and infinitely small.
3. That the ambient temperature is constant during measurement.

In reality, probes are neither infinitely long nor infinitely small. The ambient temperature of the sample is also never constant during a measurement; there is always some temperature drift.

### Better Solution

A better solution to the differential equation for finite length and radius probes can be obtained. For a heated cylindrical source of radius  $a$  (m) and length  $2b$  (m), with temperature measured at its center, the temperature rise during heating is found through equation 5.

$$\Delta T = q4\pi k \int_{\frac{a}{r}}^{\infty} u^{-1} \exp\left[-\left(\frac{a}{r}\right)^2 u\right] I_0(2au/r) \operatorname{erf}\left(\frac{b}{r}\sqrt{u}\right) du \quad (5)$$

Here,  $I_0(x)$  represents a modified Bessel function of order zero,  $\operatorname{erf}()$  is the error function, and  $u$  is an integration variable. The quantity  $\exp\left[-\left(\frac{a}{r}\right)^2 u\right] I_0\left(\frac{2au}{r}\right)$  approaches unity as  $a/r$  approaches 0, and  $\operatorname{erf}\left(\frac{b}{r}\sqrt{u}\right)$  approaches unity as  $b/r$  approaches infinity. In these limits, eq. 5 becomes eq. 1.

### Simplified Approximation: How Does It Work?

Use Equation 5 to assess the errors which can arise by using eq. 1 or eq. 4 to obtain values for  $k$  or when finite length and diameter probes are used. The construction of the KD2 and both the KS-1 and the TR-1 thermal needles are consistent with an assumption that the source radius,  $a$ , and the measurement radius,  $r$  are the same. The probe lengths and diameters suggested by IEEE 442-03 and the dimensions Decagon’s small single needle probes are given in Table 1. ASTM 5334-14 does not specify

a needle size.

	IEEE Field	IEE Lab	KS-1
Length(mm)	2000	100	60
Diameter (mm)	8	2.4	1.27

Table 1. Needle dimensions suggested by IEEE 442-03 and the Decagon KS-1 and TR-1 needle dimensions. The KS-1 best matches the assumptions of the infinite line heat source theory and measures with good accuracy without calibration.

Simulated temperature rise data were generated using eq. 5 and then used in eq. 4 to compute an apparent value for  $k$ . The difference between conductivity predicted from eq. 4 and Figure 1 shows the true conductivity (used to generate the temperature data) for the three probe diameters. Since the slope of the  $\ln$  plot changes somewhat with time the error also changes with time. The time scale shows the center time at which the slope in eq. 4 was computed. Values for three probe sizes and two thermal conductivities are shown.

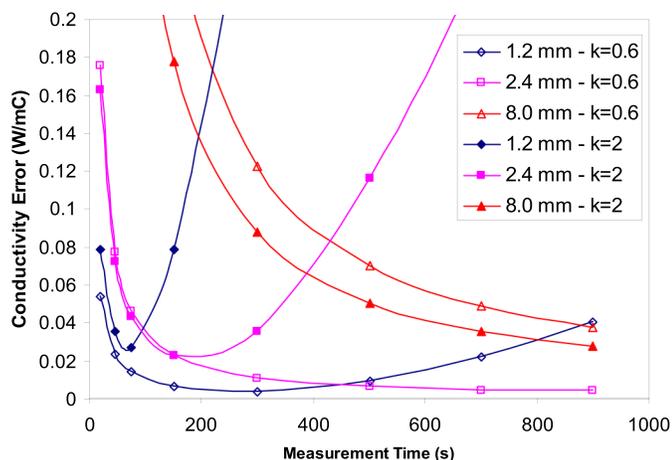


Figure 1. Error in the  $k$  value computed using eq. 4 as a function of the time at which the slope is computed for the probes shown in Table 1. This information is for uncalibrated probes. Properties for the simulation are moist soil and saturated quartz sand.

### Conclusions from Figure 1

Three things are readily apparent from the figure. First, probe size strongly affects error. The larger the probe, the larger the error at a particular time. Second, errors decrease with time, so that even large probes have acceptably small error after sufficiently

long time. The third observation is that error starts to increase after sufficient heating time. This is due to the finite probe length. For an infinitely long probe, the error continues to decrease with time. All of the probes in Table 1 are sufficiently long to give negligible error from finite probe length if the reading is taken at the proper time. The absolute error appears to be independent of conductivity of the medium, so fractional error will be larger in low conductivity samples than in high. It is important to remember that the errors shown in Figure 1 only occur if these probes are not calibrated (raw response used to compute  $k$ ). Calibration of the probes with thermal conductivity standards eliminates this type of error.

The effect of finite probe diameter on measurement error is always an overestimation of conductivity or an underestimation of thermal resistivity. All of the errors shown in Figure 1 are easily eliminated by calibrating against standards of known thermal conductivity, but probes are often used without calibration. By using the large probe specified by IEEE 442-03 without calibration, thermal conductivities and resistivities can be in error by 30 to 50% in some cases.

### Limitations of Long Heating Times

Except for the field probe, acceptable error values appear to be obtained after 30 to 200 s heating. However, long heating times are detrimental in at least two ways. In moist soil, water moves from regions of high temperature to regions of low temperature. The heating of the needle therefore drives moisture from around the needle. This reduces the thermal conductivity in exactly the region where conductivity is being measured. Minimizing heating time reduces the magnitude of this error.

The second effect of long heating times on error is through the effect of sample temperature drift on the results of the measurement. The method proposed in the IEEE standard is extremely susceptible to sample temperature drift during the measurement time.

Figure 2 shows the effect on error of an extremely small sample temperature drift of 0.001 °C/s. Error, in

this case, is the difference between the probe reading with temperature drift and the reading without, divided by the reading without. Error is minimized by using short heat times, since the probe heats very little at long times and the effect of drift is relatively larger then. The conductivity of the material affects the error because the temperature rise is of the needle is smaller in the higher conductivity material than in the low, so the relative effect of the drift is larger. This temperature drift error is a result of analyzing just the heating phase during the probe measurement. The KD2 Pro analyzes both the heating and the cooling phases. The analysis then eliminates this source of error.

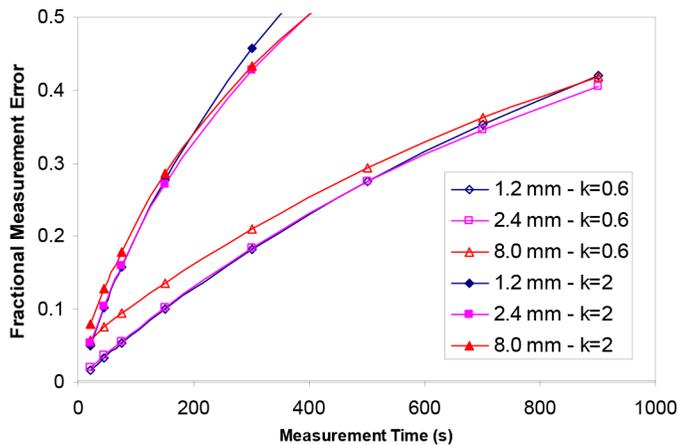


Figure 2. Error in computing  $k$  as a function of time of slope measurement for the probes in Table 1 when the temperature drift in the material under test is  $0.001\text{ C/s}$  for two thermal conductivities.

With that background, we can now compare details of the KD2-Pro measurement and analysis to those of the standards. The smaller KS-1 needle best matches assumptions of the infinite line heat source theory. The larger TR-1 needle conforms to specifications for the IEEE 442-03 lab needle. Figures 1 and 2 indicate that the smaller needle gives better results if eq. 4 is used for the analysis. In fact, if calibrations are done against standards of known conductivity, both of the smaller needle sizes will give accurate results.

### Limitations of IEEE “Field Needle”

IEEE 442-03 calls its large needle a “field needle,” but

never indicates why or where one would use such a probe. The so called laboratory needle is not only much more accurate, it is much easier to use in the field. Apparently the authors envisioned a hole for the probe being prepared using a steel pilot rod with a slide hammer. Such an insertion method would severely compact the soil around the probe, thus rendering the measurements invalid. In addition to problems with the installation, there are problems with the construction and analysis. The heater inside the probe consists of a single wire buried in epoxy. The epoxy, temperature configuration inside the probe, and stainless steel making up the probe all influence the measurement. Such a device might be susceptible to analysis using numerical models and suitable numerical inversions, but certainly is not suitable for analysis using eq. 4, and possibly not even eq. 5. If measurements are desired at depth one can easily auger a hole and place the “laboratory” probe in the soil at the bottom of the hole. This avoids compacting the soil and gives an accurate measurement at the location desired. We see no reason to ever use the field needle.

### Analysis: By Hand or By Computer?

The IEEE standard suggest collecting data with pencil and paper over a 1000 or 600 s heat time, plotting the data on semi-log graph paper, selecting a segment of the data by eye that appears to fit a straight line, selecting two points on that line to enter into eq. 4, and computing  $\rho$  from the reciprocal of eq. 4. The KD2 Pro collects data at 1 s intervals during a 30 s heating time and a 30 s cooling time. Temperature is measured by a 24 bit A to D converter. The exponential integral solution is fit to all of the data for both heating and cooling by non linear least squares, while removing effects of temperature drift during the measurement. All of the computations are done by an internal 16 bit microcontroller, and the result during the measurement. All of the computations are done by an internal 16 bit microcontroller, and the result is displayed. Because all the computations are done internally, there is no need to record individual temperature values, forty data points are used to determine the value of  $k$  rather than just 2, linear temperature drift effects

are removed, and subjectivity inherent in manual or “eye” fitting of data is removed. The accuracy of the measurement is verified using thermal conductivity standards such as glycerol and agar-stabilized water whose conductivity is known.

**KD2 Compliance to IEEE 442-03**

Decagon’s claim that the KD2-Pro conforms to IEEE 442-03 is based on the fact that the KD2 Pro uses a transient line heat source or transient heated needle method and finds thermal resistivity using an approximation to the solution to the differential equation for an infinite line heat source. The TR-1 needle matches the specifications of the IEEE lab needle. Analysis is done within a microcontroller, as a result of an improved understanding of the physics, and improvements in technology that have taken place since this standard was initially authored.